Intersections of Convex Bodies

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ABSTRACT

The Brunn-Minkowski theory is the core of analytic convex geometry, and for over a century it has provided the tools for understanding many fundamental issues concerning the metrical properties (such as volume, surface area, mean width, etc.) of convex bodies. It is particularly successful when dealing with Minkowski (vector) sums of convex bodies or their orthogonal projections onto subspaces, but it does not seem so generally applicable to intersections of convex bodies.

Several natural functions involving intersections of convex bodies can be defined, and some have been studied before. After an introductory discussion of these and some applications, we focus on two such functions: The function $\alpha_K(L, \rho)$ that gives the volume of the intersection of one convex body K in $\mathbb{R}^n$ and a dilatate ρL of another convex body L in $\mathbb{R}^n$, and the function $\eta_K(L, \rho)$ that gives the $(n-1)$-dimensional Hausdorff measure of the intersection of K and the boundary $\partial(\rho L)$ of ρL. The main interest is in the concavity properties of $\alpha_K(L, \rho)$, particularly when K and L are symmetric with respect to the origin. The Brunn-Minkowski inequality can be applied, but does not always yield the best results, which exhibit an interesting change between low and higher dimensions.

The new results discussed in the talk were obtained jointly with Stefano Campi and Paolo Gronchi of the University of Florence.