In this talk we consider essential combinatorial questions on the intersection of tropical hypersurfaces, which are given by tropical polynomials $g_1, \dots, g_k$ in n variables with Newton polytopes $P_1, \dots, P_k$. In particular, we study the f-vector, the number of unbounded faces and (in case of a curve) the genus.

The tropical questions lead to fundamental connections between certain mixed volumes and alternating sums of integer points in Minkowski sums of polytopes. By establishing some aspects of a mixed version of Ehrhart theory we can show that the genus of a tropical intersection curve equals the genus of a toric intersection curve corresponding to the same Newton polytopes.

(Based on joint work with Reinhard Steffens, arXiv:0902.1072)